Reg.No. ____________

Karunya UNIVERSITY

(Karunya Institute of Technology & Sciences)

(Declared as Deemed-to-be University under Sec.3 of the UGC Act, 1956)

End Semester Examination – Nov/Dec – 2016

			Semester	:	**2016-17 ODD**
Code	:	**14MA3005**	**Duration**	:	**3hrs**
Sub. Name :		**Calculus of Variations and Vector Spaces**	**Max. marks :**		**100**

ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)

Q. No.	Sub Div.	Questions	Course Outcome	Marks
1.	a.	State and prove Brachisto Chrone problem.	CO1	12
	b.	Find the extremal of the functional $\int_{x_0}^{x_1} \frac{\sqrt{1+y'^2}}{y} dx$	CO1	8
		(OR)		
2.	a.	Find the plane curve of fixed perimeter and maximum area.	CO1	10
	b.	Find the extremal of the functional $I = \int_0^1 (1+y''^2)dx$ given that y(0) = 0, y'(0)=1, y(1) = 1, y'(1) = 1.	CO1	10
3.	a.	Find the differential equation corresponding to the integral equation $y(x) = \int_0^x t(t-x)y(t)dt + \frac{x^2}{2}$	CO2	10
	b.	Find the integral equation corresponding to the differential equation $y''-5y'+6y=0$, y(0) = 0, y'(0) = 1.	CO2	10
		(OR)		
4.	a.	Using the method of successive approximations, solve the integral equation $y(x) = x - \int_0^x (x-t)y(t)dt.$	CO2	10
	b.	Find the characteristic numbers and eigen functions for the homogeneous integral equation $y(x) - \lambda\int_0^1 (2xt-4x^2)y(t)dt = 0.$	CO2	10
5.	a.	Prove that union of two subspaces is a subspace if and only if one is contained in the other.	CO3	10
	b.	Prove that union of two subspaces need not be a subspace.	CO3	3
	c.	Prove that arbitrary intersection of subspaces is a subspace of a vector space V.	CO3	7
		(OR)		
6.	a.	Are the vectors α_1 = (-1,2,-2), α_2 = (1,2,1), α_3 = (-1,-2,0) linearly independent in R^3?	CO3	5
	b.	Find out values of k for which the set S of vectors (3,1,2), (-2,k,5), (19k,18,19k) is not a basis of R^3.	CO3	10

	c.	Verify whether $w = \{\alpha = (a_1, a_2, a_3, ..., a_n) \in R^n \mid a_2 = a_1^2\}$ is a subspace or not.	CO3	5
7.	a.	Let V be an inner product space. Then prove that (i) $\|c\alpha\| = \|c\| \|\alpha\|$ (ii) $\|\alpha\| > 0$ for $\alpha \neq 0$ (iii) $\|(\alpha \mid \beta)\| \leq \|\alpha\| \|\beta\|$ and (iv) $\|\alpha+\beta\| \leq \|\alpha\| + \|\beta\|$ for $\alpha, \beta \in V$.	CO3	2 0
(OR)				
8.	a.	Let V be the set of all polynomials of degree ≤ 2 together with the zero polynomial. V is a real inner product space with inner product defined by $(f \mid g) = \int_0^1 f(x)g(x)dx$. Starting with the basis $\{1, x, x^2\}$, obtain an orthonormal basis for V.	CO3	1 0
	b.	Apply the Gram-Schmidt process to the vectors $\beta_1 = (3,0,4)$, $\beta_2 = (-1,0,7)$, $\beta_3 = (2,9,11)$ to obtain an orthonormal basis of R^3 with the standard inner product.	CO3	1 0
		<u>Compulsory:</u>		
9.	a.	Find the inverse Z-transform of $\dfrac{2z^2 + 3z}{(z+2)(z-4)}$	CO1	1 0
	b.	Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = 0$, $y_1 = 0$ using Z-transform.	CO1	1 0

ALL THE BEST